Notes on sediment transport modeling in ROMS
* DRAFT version 4 *

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1 Introduction

Coastal embayments along the Southern Californian coast such as the Santa Monica Bay (SMB) and the Monterey Bay (MB) are exposed to a variety of inputs due to human activities such as sewage and industrial point sources and urban and agricultural nonpoint inputs. These inputs can affect the marine ecosystems of the bays and may lead to accumulation of contaminants in the deposited sediments. The objectives of the sediment transport modeling effort is to develop a numerical model to compute water and sediment quality and carbon cycling problems for coastal areas such as SMB and MB. The aim is to calculate transport, dispersion, resuspension and deposition of (contaminated) sediments from either point or nonpoint sources (rivers, sewage outfalls, dump sites) mainly under eventlike conditions (high winds, storms, specific spills). These events, generally typified by both high freshwater runoffs and highly energetic, significant wind waves and swell, appear to be most relevant since they provide the conditions under which most of the sediment and contaminants are input (by storm water and river water runoff) and they are the conditions under which the resuspension and transport rates of deposited sediments are largest. The focus of this model study is on the large spatial scale (basin scale $\geq L \geq 1$ km, say) and a time scale of hours to days (the duration of the event).

Sediment patterns on Southern Californian shelves are typically: medium-coarse grains very near the shore and river mouths ($<$ 20 m depth), transport presumed to be dominated by littoral drift; medium-fine sand covered by fine silt deposited after winter storms on the inner shelf ($20 < h < 50$m). Observations by Drake *et al.* (1985) suggest that on the San Pedro shelf, for example, the tidal motion alone is generally not strong enough to resuspend sediments into the water column. The stirring action of waves keeps the silt in suspension in a nepheloid layer of up to 10 m thick. Under fair weather conditions, the tidal and subtidal motion is thought to redistribute this material over the inner shelf very gradually. Measurements of sand ripples on the San Pedro Shelf by Drake *et al.* (1985) suggest that the bottom deeper than about 30 to 40 meters is inactive under those fair-weather conditions. The authors suggest that during storm or large swell conditions, however, the silt is suspended into entire column, enhancing (offshelf) transport significantly. Then also the coarser fractions may get suspended. Hence, the texture and response of the bed is conditioned by the high energy events. Wiberg *et al.* (2002) have measured resuspension and transport of sediments over the Palos Verdes Shelf and extrapolated this using a model to cover a period of almost 20 years. They concluded that around 60 meter depth on average 10 resuspension events occured per year (lasting on average for 1.6 days); the number of events became 3 at 90 meters depth.

Gorsline *et al.* (1984) report that in winter the nepheloid layer sometimes becomes confined at its bottom by the pycnocline and hence form a mid-water maximum which can be transported more efficiently than the bottom nepheloid layer. Besides, they suggest that at the shelf break internal wave breaking may be an effective mechanism for resuspension of sediment (see also Cacchione *et al.*, 2002).

2 Model development

Regarding the time and space resolution considered, the explicit solution will refer to quantities averaged over both wind-wave and swell periods and averaged over sub-mesoscale eddy scales. One of the crucial ingredients in the sediment transport model is a reliable representation of the wave-averaged hydrodynamics and turbulence. In particular a sufficiently high near-bottom resolution will be required. The wave boundary layer will not be resolved explicitly, but the lower part of the velocity and sediment concentration profile in the current boundary layer is important for the calculation of the sediment transport rates. Similarly, an accurate assessment of the bottom boundary shear stress (including effect of waves) is required since it determines the initiation of grain motion and settling and resuspension of suspended load. Thus, the sediment concentration and current velocity profiles in the unresolved part of the near-bottom layer have to be parameterized.

Characterization of the sediments (mainly density and grain size, making general assumptions about shape and cohesiveness) is done either as a time-dependent prescribed function at the point sources or at the sea bed as an initial (space-dependent) condition. other biogeochemical interactions contamination) can be incorporated) Along with bed sediment properties also characteristics of bed form roughness will be provided as initial input. Sediment concentration may be considered as passive with respect to the flow density or as active if concentration values require such. Effects of stratification on the turbulence due to the suspended sediment may be considered in the bottom boundary layer formulations (see below).

In the following subsections the major aspect of sediment-transport relevant for Southern Californian shelves are discussed and recommendations on the implementation in ROMS are given.

2.1 Noncohesive suspended sediments

Consider n (say $n = 2$ to start with) size classes (j). For each class, the balance of suspended sediment mass per unit volume in the explicitly solved flow (i.e., for the wave-averaged concentration $c(x, y, z, t)$) reads:

$$\partial_t c_j + \nabla \cdot F_j = Q_j^E + Q_j^I \tag{1}$$

where Q^E represents all external sources and sinks (point, nonpoint), Q^I represents internal sources and sinks such as reactive decay or generation of or exchange between size classes. For brevity, index j will be omitted from now on; unless specified otherwise, sediment-related values are class-dependent. Flux F is supposed to consist of three parts:

$$F = F_a + F_d + F_s \tag{2}$$

where:

- F_a is advective flux due to the explicitly solved flow;
- $F_d = -(K_{sh}\nabla_h c, K_{sv}\partial_z c)$ eddy-diffusive flux, due to correlations of sub-filter-scale fluctuations (genuine 'turbulence', eddies and wave fluctuations). K_{sh}, K_{sv} are eddy diffusivities for sediment (size-class independent). In the interior and in the free surface boundary layer the eddy diffusivities are taken to be identical to the ones used for the transport of momentum in the model. The determination of K_{sv} in the bottom boundary layer will be further discussed in section 2.4;
- $F_s = (0, 0, -w_s c)$ gravitational settling flux; w_s is empirical value of settling velocity, dependent on sediment properties, assumed to be independent of flow conditions, separate constant for each size class. We assume uniform grains per size class, and ignore hindered settling. On our applications concentrations are expected to be low enough to to consider w_s as a constant, independent of concentration (for $c < 25$ g/l, an error in w_s of less than 5% is made in the estimate (Hamrick, 1999)).

Boundary conditions for the concentration equation are provided as follows.

Lateral boundary conditions

- Submerged vertical walls (if any) may be assumed to be non-erosive, non-depositional (i.e. $F = 0$), since the sediment is considered to originate either from the bed or from designated sources.
- Coastline boundary: since the model will not resolve the surf zone explicitly, a description of the exchange of sediment over the coastward model boundary may be required. Possible choices are either a no-flux condition or (if observations suggest so) a Neumann condition giving either a prescribed diffusive flux or a diffusive flux which depends on macroscopic (mainly wave) conditions to simulate (subgrid) exchange of sediment with the near-shore region. At this moment it is yet unclear what the nature of the exchange between near shore region and inner shelf is. A constant diffusive flux approach yields a parameter for which model sensitivity can be investigated. Fredsøe and Deigaard (1992); Nielsen (1992) give extensive discussion on near-shore processes. Rakha (1998), for example, has developed a quasi-3D model for the near-shore region which might accomplish the more offshore studies here.

  Observations on the San Pedro Shelf (Gorsline *et al.*, 1984; Drake *et al.*, 1985) suggest that the fine fraction is directly transported out of the near-shore zone and the coarse fraction remains in this zone. If the focus of the modeling studies will be on the fine sediment fractions only, it is reasonable to assume a negligible exchange of the fine material between surf zone and inner shelf except for those locations where an explicit input of fines at the model boundary is prescribed. This is, for example, also done by Zhang *et al.* (1999) on the Northern Californian shelf off Eureka. In Monterey Bay, however, Storlazzi and Field (2000); Xu *et al.* (2002) suggest that not only rivers but also cliff erosion is a significant source of fine sediments. In that case a diffuse sediment flux passing the breaker zone may have to be considered.

- Open-ocean boundary, treated as for other tracers; on outflow: radiating, on inflow: nudging to fixed value or use the method of characteristics (possible e.g. for tidal excursions but this still leaves subtidal advective flux unresolved on inflow). It may be reasonable to assume that, far offshore, the nudging value of the suspended sediment concentration approaches zero.

Vertical boundary conditions

A the top of the water column, no flux through the boundary is imposed; at the bottom an exchange flux S is specified:

$$K_{sh}\nabla_h c \cdot \nabla_h \zeta - K_{sv}\partial_z c - w_s c = 0 \quad \text{at } z = \zeta \tag{3}$$
$$K_{sh}\nabla_h c \cdot \nabla_h h - K_{sv}\partial_z c - w_s c = S \quad \text{at } z = -H + h(x,y,t) \tag{4}$$

The exchange flux ($S = E - D$) is defined as the net flux due to erosion (resuspension) and deposition on the bed, positive into the water column. The deposition rate can be written as $D = P_d w_s c$. The horizontal and vertical eddy diffusivities for sediment (K_{sh}, K_{sv}), erosion rate (E) and deposition probability (P_d) are to be determined below.

As usual, the fluxes at the water-bed interface are hypothesized as being controlled by competition between shear stress exerted by the flow on the grains (skin friction) trying to lift particles and the submerged weight of the particles. Under stationary, uniform conditions, an equilibrium distribution of suspended sediment tends to be established in the water column. Then, resuspension flux and deposition flux cancel each other. Under a number of simplifying assumptions an explicit expression can be found for the equilibrium concentration c_{eq}, either under current-only conditions (e.g., Dyer, 1986) or under combined wave-current conditions (e.g., Styles and Glenn, 2000). The wave-averaged equilibrium is given by the abovementioned balance of fluxes:

$$w_s c_{eq} + K_{sv}\partial_z c_{eq} = 0 \tag{5}$$

Depending on the chosen formulation for the dependence of K_{sv} on z, a specific c_{eq} profile may be determined. To illustrate this, a linear diffusivity profile is assumed, as is appropriate very near the bed : $K_{sv} = \kappa u_* z$. Here K_{sv} equals the linear K_v profile (i.e., Prandtl-Schmidt number = 1, as is usually done, and stratification is assumed to be negligible); κ is Von Karman's constant, u_* the friction velocity Using this, the so-called Rouse profile for c_{eq} is obtained:

$$c_{eq}(z) = C_r(\frac{z}{z_r})^{-R} \tag{6}$$

with $R = w_s/\kappa u_*$ the Rouse parameter, and C_r the 'reference concentration' at a near-bed reference level $z = z_r$ (usually equal to the physical bottom roughness length z_0).

For combined wave and current conditions a parameterization for K_v and K_{sv} has been presented by Styles and Glenn (2000), an extension of the Glenn and Grant (1987) model which was based on Grant and Madsen's (1979) bottom boundary-layer model. Styles and Glenn (2000) found a c_{eq} profile in the near-bed wave boundary layer similar to (6), albeit involving a modified Rouse parameter and a modulation of the vertical structure to incorporate stratification effects. (See section 2.4.2.)

To obtain an expression for the net exchange rate S at the bottom for non-equilibrium situations, it is posed that this flux can be parameterized as proportional to the difference between the actual concentration and the reference concentration at the reference level:

$$S = w_s(C_r - c_{|z=z_r}) \tag{7}$$

Which reflects the notion that when near-bed concentration exceeds the equilibrium value, net deposition occurs and vice versa. The reference concentration C_r can be determined from a range of (semi)empirical relations in the literature. The expressions by Smith and McLean (1977) and Van Rijn (1984b) have been reviewed and compared against observational data by Garcia and Parker (1991) and turned out to perform best. Both parametrizations amount to:

$$C_r \sim T^a \quad \text{for } T > 0 \tag{8}$$
$$C_r = 0 \quad \text{for } T \leq 0 \tag{9}$$

in which $T = (\tau_b - \tau_{cs})/\tau_{cs}$ the normalized excess shear stress, τ_b being the actual skin friction shear stress (magnitude), τ_{cs} is an empirical critical threshold stress for suspension to occur; a is an empirical power of order unity (depending on chosen parametrization).

The parametrization of C_r presently implemented in ROMS is the one proposed by Smith and McLean (1977) for steady flow conditions (also applied in recent shelf models by Cookman and Flemings, 2001; Li and Amos, 2001, for example):

$$C_r = \rho_s C_b \frac{\gamma_0 T}{1 + \gamma_0 T} \approx \rho_s C_b \gamma_0 T \tag{10}$$

where ρ_s is the sediment grain density and C_b is the volumetric concentration of the particular sediment class in the bed and γ_0 is an empirical resuspension parameter [1]. Here we choose a constant γ_0 for the time being, its actual value may be best determined by relying on field experiments (or even tuning to observational data) in the regime and area of interest. The relatively small order of magnitude of γ_0 justifies the approximation on the right hand side of (10).

Following Glenn and Grant (1987), many authors (e.g. Soulsby, 1997; Zhang *et al.*, 1999; Harris and Wiberg, 2001; Li and Amos, 2001) pose that an adapted form of the Smith and McLean (1977) formulation (10) be used to determine the reference concentration under combined current and waves. Most of them (except Zhang *et al.*, 1999) replace the current excess stress T by the excess stress due to *maximum* skin friction under combined waves and currents to obtain a representative wave-averaged reference concentration. This approach assumes that the response time of the sediment profile to develop in the lower part of the boundary layer is short compared to the wave period (i.e., no settling lag). Grant

[1] Smith and McLean (1977): $\gamma_0 = \text{const.} = 2.4 \cdot 10^{-3}$, Drake and Cacchione (1989): $10^{-5} \lesssim \gamma_0 \lesssim 10^{-3}$, depending on T and bed forms; Madsen *et al.* (1993): $\gamma_0 = 0.4 \cdot 10^{-3}$ for near-shore sheet flow; Xu *et al.* (2002): $\gamma_0 = 0.3 \cdot 10^{-3}$ for Davenport-Monterey shelf under storm conditions

and Madsen (1982) suggest that this may be a reasonable assumption. It must be noted that this approach is chosen mostly because of the lack of any better alternative. Tuning of the reference concentration to actual field observations by adjsuting the resuspension parameters γ_0 might be required.

Recommendation

For ROMS it is recommended to solve the wave-averaged suspended sediment concentration equation (1) for the appropriate grain sizes and to apply the concept of the equilibrium concentration. The reference concentration is to be computed using (10) for both the current-only or combined current conditions with substitution of the appropriate excess stresses. A continuous match between these two regimes has to be accounted for.

2.2 Bed load transport

The previous section referred to the part of the sediment transported as suspended load. This mode of transport only occurs as long as the skin shear stress exceeds a certain critical value τ_{cs}. Generally stated, very fine sediments such as silt and clay are considered to be either in suspension or at rest on the bed. However, sediments with a grain diameter larger than about 0.1 mm (i.e. very fine sand and coarser) may also be transported in a rolling and sliding fashion over the bed (denoted as bed load). In the case of the Californian shelves, which for most part consist of fine sand, silt or even finer fractions, is reasonable to neglect bed load (see e.g., Sternberg *et al.*, 1999; Xu *et al.*, 2002; Noble and Dickey, 2002).

Though, if one is considering the very nearshore region, coarser fractions become relevant. For these occassion and for completeness a outline of bedload modeling is given here. Bed load occurs when the near-bed stress is between the critical value for suspension and a critical value for the initiation of grain motion τ_{cb}. Van Rijn (1984b) discusses various criteria for the distinction between the transport modes [2]. As an upper limit for the transition to suspended load is the critical near-bed shear velocity equalling the particle settling velocity: $u_{*cs} = w_s$. A lower limit (given by Engelund, 1965) is $u_{*cs} = 0.25 w_s$. If bedload is considered it is proposed to apply $u_{*cs} = 0.8 w_s$, which is often done (e.g., Li and Amos, 2001; Cookman and Flemings, 2001) and which is in line with the intermediate range of criteria by Van Rijn (1984b). When the near-bed shear velocity is less than the selected criterium but larger than the critical shear velocity for grain motion u_{*cb} (which can be determined from the critical Shields stress value) the sediment is transported as bed load only. The Shields entrainment function θ_c is an empirical function of the grain Reynolds number R_d:

$$\theta_c = \frac{\tau_{cb}}{(\rho_s - \rho) g d} = \theta_c(R_d) \tag{11}$$

with ρ_s the sediment density, ρ the density of sea water, d the grain diameter, g the gravitational acceleration (see also e.g., Dyer, 1986).

[2] Under very energetic conditions a transition to sheet flow occurs: a suspended transport mode under which all bed forms are erased. This is mainly relevant for the determination of the bed roughness and will referred to below.

Given an instantaneous near-bed skin friction shear stress, the bed load transport can be determined from one of the many (semi-)empirical expressions are available in the literature. Generally speaking they relate the bed load transport to the shear velocity given a certain threshold shear stress.

The transport can be described using formulations based on the principles of Bagnold (1956) and Meyer-Peter and Müller (1984). They state that the (vertically integrated) bed load transport rate $\boldsymbol{q}_b$ is mainly dependent on a certain power of the excess stress at the bed:

$$\boldsymbol{q}_b \sim \psi(\tau - \tau_{cb}) \tag{12}$$

More modern parametrizations can be found in Van Rijn (1984a) and Van Niekerk *et al.* (1992) for example. The latter is an example of a formulation adapted to heterogeneous beds and includes effects of hiding and exposure of grains due to size differences. At the moment, however, it is proposed to ignore these effects for the sake of simplicity and to consider each size class independently. Van Rijns's equation is not explicitly designed for combined flow conditions, so it's success upon implementation is still to be seen. Two fairly recent models of shelf-sea sediment transport under combined wave and current conditions (Cookman and Flemings, 2001; Li and Amos, 2001) have applied the bedload equation of Yalin (1963) with apparent success. This formulation also conforms to (12) with the restriction that it is applied best when limited to grains of 0.2 mm and coarser. For each size class the critical shear stress can be determined from the critical Shields stress value and the wave-averaged transport is obtained after integrating the instantaneous transport due to the combined wave-current shear stress at the bed over a wave cycle. Another parametrization valid for combined flow conditions is the one developed by Bailard (1981).

Recommendation

For transport of the predominantly fine sediments on the Southern Californian shelves the bedload component is of minor importance It is therefore proposed to ignore bedload transport for the time being.

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2.3 Cohesive sediments

The settling, erosion and deposition for very fine (cohesive) sediments could be described using a model based on Ariathurai and Krone (1976), one of the seminal papers on which many others have elaborated. When sediment particles are cohesive the flocculation affects the settling velocity w_s. Flocculation is function of sediment properties, concentration and flow and fluid (salinity) properties; it is computationally expensive to take all these effects into account in detail. Some approximating semi-empirical parametrizations for w_s are available which depend on sediment concentration (some also on floc size) and the shear stress in the water column (e.g., Ariathurai and Krone, 1976; Hwan and Mehta, 1989).

It is proposed to start here by assuming that sediments are noncohesive and settling velocity is constant. If, though, effects typical for the very fine, partly cohesive sediment fraction are to be taken into account, a relatively simple step might be to implement modified expressions for the erosion and deposition rate as has been done by Ribbe and Holloway (2001):

$$D = \quad c w_s \frac{\tau_b - \tau_{cd}}{\tau_{cd}} \quad \text{if } \tau_b \leq \tau_{cd} \tag{13a}$$

$$D = \quad 0 \quad \text{if } \tau_b \geq \tau_{cd} \tag{13b}$$

which amounts to posing that the settling probability P_d is proportional to an excess shear stress, when compared to section 2.1 under eq. (4). The critical depositional shear stress τ_{cd} may differ from the critical erosion stress (see below) and is determined empirically (cf. Mehta *et al.*, 1989).

The determination of the resuspension flux from a cohesive bed is in general very complicated. The rate depends on the bed characteristics (rheology) and usually two types of erosion are considered: (A) rapid *mass erosion* (or bulk erosion), when the skin friction shear stress exceeds the depth-dependent yield stress (shear strength) τ_y in the top layer of the bed; (B) gradual *surface erosion*, which occurs when the skin friction shear stress exceeds the critical stress for individual particles but is less than the yield stress. Mass erosion can be expressed as inversely proportional to a mass-transfer time scale and is not discussed further here. Surface erosion is often regarded as either depth-limited ('Type I') consolidated beds) or as unlimited erosion ('Type II') appropriate for consolidated beds) (Sanford and Maa, 2001). Type I erosion is characterized by an erosion rate that depends exponentially on the excess stress:

$$E \sim \quad e^{\alpha[(\tau_b - \tau_{ce}(z))/\tau_b]^\beta} \quad \text{if } \tau_y > \tau_b \geq \tau_{ce} \tag{14}$$

in which the critical erosion stress τ_{ce} depends on the vertical position in the bed and α and β are empirical constants.

Type II erosion is generally represented by a power of the excess stress:

$$E \sim \quad (\frac{\tau_b - \tau_{ce}}{\tau_{ce}})^b \quad \text{if } \tau_y > \tau_b \geq \tau_{ce} \tag{15}$$

with constant τ_{ce} and b an empirical constant ranging from 1 to about 4 in various studies.

Sanford and Maa (2001) proposed a formulation in which these two erosion types are unified. They found that the erosion of the bed is controlled by the ratio of the time rate of change of the near-bed shear stress to the time scale of the sediment depletion. If the shear stress time scale is relatively long (such as in a tidal current) the erosion is depth limited and almost entirely controlled by the time rate of change of the shear stress (Type I erosion). If, on the other hand, the shear stress varies relatively rapidly (such as under swell waves), the erosion behavior is more like Type II and is more in phase with the forcing because there is less depletion of sediment during one wave cycle. To derive a wave-averaged expression for the erosion rate which is suited for combined wave and current conditions, a wave-averaged adaptation of the Sanford and Maa (2001) formulation might be considered.

A zeroth-order approach to the erosion of cohesive sediment might be to ignore the down-core dependence of τ_{ce} and apply eq. (15) with $b = 1$ as has for example been done by Ribbe and Holloway (2001). In our application the determination of the wave-averaged erosion rate has to be along the same lines as has been done for the reference concentration in section 2.1. The instantaneous erosion rate then would read:

$$E' = \quad M(\tfrac{\tau_b' - \tau_{ce}}{\tau_{ce}}) \quad \tau_b \geq \tau_{ce} \tag{16}$$

with M an empirical constant (of order 10^{-4} kg m^{-2}s^{-1} for low-frequency conditions, Ribbe and Holloway, 2001; Sanford and Maa, 2001). Note that $\tau_{cd} < \tau_{ce}$ so that in general a stage exists in which neither erosion nor deposition of fine material occurs.

Recommendation

It is recommended to start by ignoring cohesiveness of the sediment. If some of the first order effects are to be taken into account the noncohesive suspended sediment routine could be adapted by considering eqns. (13) and (15) or their wave-averaged counterparts as the deposition and erosion rates of cohesive sediment.

2.4 Bottom boundary layer model and shear stresses

Form the above it is clear that the exchange of suspended sediment between bed and fluid depends primarily on the shear stress exerted on the grains by the flow (skin friction). The same is true for the bedload transport rate. Hence, an accurate description of the bottom boundary layer (BBL) is essential to determine the magnitude and direction of the shear stress near the bed. It should also be noted that a difference exists between the actual skin friction and the frictional stress as experienced by the flow, which may exceed the skin friction due to the presence of bed forms (form drag) and near-bed transport of sediment (bed load or sheet flow). It is the skin friction that is repsonsible for the motion of sediment at the bed, while the form drag is associated with the turbulence that diffuses the sediment further up into the water column.

The suspended sediment itself, on the other hand, may damp turbulent motions in the near-bed region. Hence, a range of feedbacks exist between the wave-averaged flow, the turbulence and the sediment transport.

2.4.1 BBL, current only

To determine the bottom shear stress due to currents only, τ_c, it is most straightforward to apply a quadratic drag law, using the Von Karmán-Prandtl 'law of the wall' related to the resolved near-bottom current u at reference level $z = a$ within the logarithmic layer for a given roughness length z_0:

$$\tau_c \quad = \rho \left(\tfrac{\kappa}{\ln(a/z_0)}\right)^2 u^2|_{z=a} \quad = \frac{1}{2}\rho f_c u^2|_{z=a} \tag{17}$$

where $\kappa = 0.4$ is the Von Karmán constant, f_c is the current friction factor corresponding to the reference level.

The proposed sequence in determining the friction is as follows: first the skin friction is determined, then the roughness characteristics, and —if considered relevant— bedload roughness and near-bed suspended sediment concentration are determined; from this the total friction can be assessed.

In time-independent models such as of Li and Amos (2001) and in the 2DV model by Harris and Wiberg (2001), for example, these friction and roughness terms are computed iteratively. If the roughness evolves on a much slower time scale than the dominant current it might be possbile to update these terms only once per time step and project them directly to the next time level, after a few time steps this should be converged.

To determine the skin friction τ_{bs} under low-frequency (i.e., tidal or subtidal) current conditions, a constant f_c[3] is applied in eq. (17) (as is done by many, e.g., Sternberg, 1972; Soulsby, 1983; Li and Amos, 2001).

The total bottom τ_{bt} stress is parameterized similarly, but with a roughness length dependent on the ripple (or bedform) wavelength λ and height η. Often-used expressions are:

$$k_b = 27.7\frac{\eta^2}{\lambda} \tag{18}$$

$$z_0 = \frac{k_b}{30} \tag{19}$$

$$\lambda = 1000d \tag{20}$$

$$\eta = 0.074\lambda^{1.19} \tag{21}$$

in which d is the mean sediment grain diameter; eqns. (18), (20) and (21) are empirical relations stemming from Grant and Madsen (1982), Yalin (1964) and Allen (1970), respectively. The latter two only apply to current generated ripples on sandy beds.

2.4.2 BBL, combined waves and currents

As stated in the introduction, the stirring of sediments by waves and swell is presumed to be an important aspect in the sediment transport on the Southern Californian shelves. At this stage of the model development it is proposed that the relevant wave input parameters —wave direction, root-mean-square or significant wave height (H_{rms}, H_s resp.) and peak period (T_p, angular frequency ω)— will be determined in an 'off line mode' from a separate swell-wave model (cf. O'Reilly and Guza, 1993, see also `http://cdip.ucsd.edu`)[4]. The wave parameters will be fed into ROMS as dependent on space, and possibly later also time. At each location the parameters required to determine near-bed shear stress (mainly near-bed orbital velocity and excursion amplitude) are determined using linear wave theory, given the local depth. It thus is assumed that waves are linear (and certainly not breaking)

[3]Values are listed by Soulsby (1997), for example: silt/sand: z_0 = 0.05 mm, f_c=3.2 10^{-3}; mud/sand: z_0 = 0.7 mm, f_c=6 10^{-3}.

[4]The Scripps swell model makes use of swell and wave data observed by buoys and does not take into account local generation of wind waves inshore of these buoys. For local seas, which conform to the Pierson-Moskowitz or (on shallower water) the JONSWAP spectrum, estimates of significant wave height and peak period have to be obtained from other local observations and/or models.

and that wave-driven currents may be neglected. Also, feedback from the current on the waves is neglected.

Under combined wave-current conditions the boundary layer dynamics are quite complex. Styles and Glenn (2000) present a boundary layer model for combined flow conditions based on the concepts of Grant and Madsen (1979) and extensions of this work by Glenn and Grant (1987). In their model, the wave BBL is split into two parts to ensure a continuous profile of the eddy viscosity and diffusivities over the entire combined boundary layer:

$$K_v = \kappa u_{*c} z \quad z > z_2 \tag{22a}$$

$$K_v = \kappa u_{*cw} z_1 \quad z_1 < z \leq z_2 \tag{22b}$$

$$K_v = \kappa u_{*cw} z \quad z_0 < z \leq z_1 \tag{22c}$$

where z is defined within the boundary layer, z_1 is an (arbitrary) scale defining the lower boundary of the transition layer in which K_v is constant, and $z_2 = z_1 u_{*cw}/u_{*c}$; u_{*c} is the current-shear velocity, u_{*cw} is the factor 0.74 for rouse number? maximum combined flow shear velocity derived from the of the combined stress: $u_{*cw}^2 = C_R u_{*wm}^2$, with u_{*wm} the maximum shear velocity due to waves, and C_R being the friction enhancement factor (Grant and Madsen, 1986):

$$C_R = \left[1 + 2\left(\frac{u_{*c}}{u_{*wm}}\right)^2 \cos\varphi + \left(\frac{u_{*c}}{u_{*wm}}\right)^4\right]^{1/2} \tag{23}$$

with $0 \leq \varphi \leq \pi/2$ the angle between current and waves.

Strictly speaking, the derivation of the combined wave-current boundary layer model relies on the presumption that the bed-orbital velocity (u_b) is at least of the same order of magnitude as the current velocity near the bed (since the scaling velocity is u_b).

However, according to R. Styles (pers. comm.), Styles and Glenn, unpubl. manuscript... the model as implemented in ROMS is adjusted to be applicable to the entire range of wave-dominated to current-dominated conditions.

The Styles and Glenn (SG) bottom-boundary layer model also incorporates the effects of stratification due to the suspended sediments, by analogy with thermally stratified atmospheric boundary layers:

$$K_{v_{\mathrm{strat}}} = \frac{K_v}{1 + \beta\frac{z}{L}} \tag{24a}$$

$$K_{sv_{\mathrm{strat}}} = \frac{K_{sv}}{\gamma + \beta\frac{z}{L}} \tag{24b}$$

where $\gamma = 0.74$ and $\beta = 4.7$ are constants adopted from atmospheric boundary layer studies, L is the Monin-Obukhov length scale which in turn depends on the vertical turbulent sediment flux. Styles and Glenn (2000) close this flux with the Reynolds averaged K-diffusive flux which gives rise to an implicit expression for $K_{sv_{\mathrm{strat}}}$.

Shear velocities (u_{*c}, u_{*wm}) and reference concentration are computed to determine the piecewise wave-averaged profiles of current and equilibrium sediment concentration throughout the wave boundary layer. The SG-model also provides skin-friction coefficient and effective bottom roughness due to bed load and wave ripples along the lines of Glenn and Grant (1987).

2.4.3 Alternative approach, applicable to all conditions

The use of wave-current boundary layer models of the Grand & Madsen type requires the specification of an extensive set of parameters that may depend on local field conditions (which may be hard to assess). Besides, the vertical grid of ROMS is too coarse to resolve the wave-current boundary layer, so that a subgrid model is used, which requires extra computational effort. Therefore, an alternative approach is proposed here, based on approximations presented by Soulsby (1997). A similar simplification has been applied by Zhang *et al.* (1999), for example. In this way a parametrization for the combined wave-current bottom stress is obtained in which the maximum combined stress matches the current-only-induced stress when the wave motion vanishes, so that also one parameterization for the reference concentration suffices.

In a model intercomparison study Soulsby (1995) derived a two-coeffcient optimisation to the four most succesful widely applied wave-current BBL models at that moment (among which Grant and Madsen's). He expressed the enhanced bed-shear stress under combined conditions averaged over a wave cycle ($\bar{\tau}_{cw}$) in terms of the shear stresses which would occur due to the waves alone (τ_w) and due to the currents alone (τ_c):

$$\bar{\tau}_{cw} = \tau_c \left[1 + 1.2 \left(\frac{\tau_w}{\tau_c + \tau_w} \right)^{3.2} \right] \tag{25}$$

The computation of $\bar{\tau}_{cw}$ is also done in two stages to compute the skin and total friction as outlined in section 2.4.1; τ_c is determined from eq. (17) using a constant f_c, τ_w is determined from the standard expression for purely wave-induced bottom friction:

$$\tau_w = \frac{1}{2} \rho f_w u_b^2 \tag{26}$$

where u_b is the bottom orbital velocity and f_w is the wave friction factor for which a range of expressions is available in the literature.

$$f_w = 1.39 \left(\frac{A_b}{z_0} \right)^{-0.52} \tag{27}$$

where $A_b = u_b/\omega$ is the wave orbital excursion amplitude.

The combined skin friction is determined using grain roughness for z_0. The maximum stress τ_{cw} due to the combined flow can be determined from vectorial addition of $\bar{\tau}_{cw}$ and τ_w:

$$\tau_{cw} = [(\bar{\tau}_{cw} + \tau_w \cos\varphi)^2 + (\tau_w \sin\varphi)^2]^{1/2} \tag{28}$$

Once the maximum skin friction is determined, the bed roughness and reference concentration can be calculated. the actual stress felt by the sediment. The turbulence in the flow and hence the vertical velocity profile of sediment concentration depends on the total combined stress including from drag. For this, z_0 in both eqns. (17) and (26) is replaced by the apparent roughness (see below). For a flat bottom and with disregard of any bedload roughness (as is most often done for silt and clay), total friction equals skin friction. Using the friction velocities derived from the total shear stress ($\bar{u}_{*cw} = \sqrt{\bar{\tau}_{cw}/\rho}, u_{*cw} = \sqrt{\tau_{cw}/\rho}$) the vertical profile of velocity $u(z)$ in and just above the wave boundary layer is given by:

$$u(z) = \frac{(\bar{u}_{*cw})^2}{\kappa u_{*cw}} \ln(\frac{z}{z_0}) \quad z_0 \leq z \leq \delta_{cw} \tag{29a}$$

$$u(z) = \frac{\bar{u}_{*cw}}{\kappa} \ln(\frac{z}{z_a}) \quad z > \delta_{cw} \tag{29b}$$

with $\delta_{cw} = u_{*cw}/\omega$ the wave boundary layer under combined forcing and $z_a = \delta_{cw}\left(\frac{z_0}{\delta_{cw}}\right)^{\frac{\bar{u}_{*cw}}{u_{*cw}}}$ such that in the limit of vanishing wave motion the velocity profile reduces to the log profile for pure currents ($u_{*cw} \rightarrow \bar{u}_{*cw} \rightarrow u_{*c}$). The equilibrium concentration profile of suspended sediment is:

$$c(z) = C_r\left(\frac{z}{z_0}\right)^{\frac{-w_s}{\kappa u_{*cw}}} \quad z_0 \leq z \leq \delta_{cw} \tag{30a}$$

$$c(z) = C(\delta_{cw})\left(\frac{z}{\delta_{cw}}\right)^{\frac{-w_s}{\kappa \bar{u}_{*cw}}} \quad z > \delta_{cw} \tag{30b}$$

with C_r the reference concentration (cf. eq. 10) with the excess stress now depending on the maximum combined skin friction at the bed: $T = (\tau_{cw_{skin}} - \tau_{cs})/\tau_{cs}$. For vanishing waves the concentration profiles reduce to the current-only profile of eq. 6. Equations (29) and (30) can be used to determine the velocity and equilibrium concentration at the lowest grid cell (outside the wave boundary layer).

For silty beds the only roughness elements of significance are biogenic mounds and burrows (Harris and Wiberg, 2001). Their spacing (λ_{bio}) and height (η_{bio}) is to be specified on input. Equation (18) may be used to determine the roughness height. Harris and Wiberg (2001) provide an expression for the decay of biogenic bed forms due to wave-induced shear stress which might be considered at a later stage.

Recommendation

It is recommended to follow a as simple as possible approach to the modeling of the bed shear stress and its related quanitites near the bed. A parametrization which is valid for either wave, current or combined conditions is provided by the expressions due to Soulsby (1997) and is outlined in section 2.4.3. In this approach stratification effects of the sediment are neglected, the combined shear stresses are derived from a two-coefficient empirical expression in terms of pure-wave and pure-current stresses. Moreover, since the focus is on fine-grained material, the form drag is assumed to be only due to specified biogenic roughness elements. It is proposed to test this method and compare it to the much more elaborate model by Styles and Glenn (2000).

Flux condition for transprt eqn sediments assumes that entrainment rate can indeed be expressed in terms of reference (stationary equilib) conc SmithMcLean per fraction (constant porosity) applied at reference height under non-stationary conditions: $F = f_i w_s C_a$

Mass fractions f_i computed assuming conservation of mass in each fraction (no interaction of fractions), constant porosity (bed conc.) (no compaction or consolidaton).

Turbulent eddy diffusivity Ksediment = Ksalt.

2.5 Bed model

A solution of the mass balance of the bed is required as soon as net erosion or deposition become significant with respect to the total amount of erodible sediment on the bed. As long as sediment layer properties are considered as time independent due to cohesion, benthic life etc., it is relatively straightforward to model the accretion or depletion of the bed. It is therefore proposed to disregard biogeochemical processes in the bed at first and to just keep track of the amount and composition (in terms of size classes) of the sediments in the bed for a given volumetric bed concentration C_b. The time rate of change of the bed level equals the horizontal divergence of the sediment flux:

$$C_b \partial_t h = -\nabla_h \cdot \boldsymbol{q}_b - \frac{S}{\rho_s} \tag{31}$$

which, with respect to the suspended load, is what is presently done in the sediment-transport version of ROMS.

In areas with limited supply of sediments, the depletion of the bed may become limited in time. Either the sediments can be totally removed, leaving the non-erodible bedrock exposed, or selective entrainment (winnowing) of the fine fraction can cause 'bed armoring', the coarser fraction preventing the bed from being eroded further. Zhang *et al.* (1999) and Harris and Wiberg (2001), for example, discuss the modeling of an active bed layer with some degree of armoring.

2.5.1 Armoring, active layer, stratigraphy

In bed of mixed composition the erodability of the individual fractions depends on the composition of the total mixture in the layer exposed to shear stresses. The erosion rate of the smaller than median fraction in a mixture is usually reduced compared to a uniform bed of that grain size due to *hiding*. Conversely, the coarser grains get *exposed* in a mixed bed and erode more easily than under uniform conditions. Armoring occurs when the fine fraction in the surface layer gets winnowed and a covering layer of coarse-grained material develops and starts to inhibit further resuspension of the fine fractions that lie inbetween and underneath the coarser grains.

Several studies indicate that armoring is an important process on the Californian shelves Drake and Cacchione (1989); Harris and Wiberg (2002); Reed *et al.* (1999) and several approaches have been followed to model this in recent years. All involve a so-called 'active layer' on top of a substrate (see Armanini, 1995, for a formal discussion). In this relatively thin layer the sediments are reworked by the flow and thus are available for resuspension. Thickness δ_a of this layer is of a few millimeters and is expressed either in terms of grainsize diameters ($3d_{90}$, often done in fluvial applications), or as a fixed thickness (mainly in shelf applications such as Reed *et al.*, 1999; Zhang *et al.*, 1999).

Here we choose a simple approach of a layer of constant thickness (Reed *et al.*, 1999, $\delta_a = 0.3$ cm). on top of a substrate. The vertical structure of the substrate is assumed to be well mixed. Hence the bed consists of an active layer and one substrate layer. The composition of the substrate varies over time depending on the exchange of the sediments between bed and water through the active layer. The rationale of this 2 layer apporach is that at the moment we aim to study only short-term events in which the development of the stratigraphy below the active layer is of secondary importance.

The hiding and exposure is modeled according to Garcia and Parker (1991). In this approach two factors affect the erosion rate of each fraction: the distance of the particular size class from the median in the size distribution and the spread (straining) in the sediment distribution (the wider the distribution, the stronger the hiding and weaker the exposure). Garcia and Parker (1991) developed the hiding-exposure term as an empirical correction factor depending on size class i (diameter d_i, fraction f_i) to an entrainment rate for uniform sediment of that size class (E_u):

$$E_i = \zeta_i E_u = \left(\frac{d_i}{d_{50}}\right)^{1.0} \lambda_E^5 E_u \tag{32}$$

in which d_{50} is the median diameter and $\lambda_E = 1 - 0.29\sigma_\phi$ a 'straining parameter' depending on the standard deviation ($\sigma_\phi^2 = \sum_{i=1}^n (\phi_i - \bar{\phi}) f_i$) on the sedimentological ϕ-scale([5].

Garcia and Parker (1991) derived the expression from riverine data, but is has been successfully applied in marine environments for example by Walgreen *et al.* (2003). We will apply this correction term to the non-cohesive and cohesive entrainment rates of our choice (eqns. (10) in (7), and (16), respectively).

Recommendation

It is recommended to solve the mass balance for the bed using (31), given the net erosion/deposition of suspended sediments S, disregarding bedload fluxes. For the time being, only limitation by the total amount of sediment will be considered by specifying the initial amount of sediment in the active layer and keeping track of the mass in this layer over time.

3 Proposed experiments

It is proposed to first set up a simplified test model representing a typical Southern Californian coastal embayment. In this way we may get more insight in the considered sediment transport processes themselves as well as in the modeling thereof while saving computational effort. The choice is to idealize the geometry of the shelf: e.g. piecewise linearly sloping bottom, enclosed by straight coastlines, and to apply simplified forcings (one major tidal constituent, unidirectional constant wind, uniform wave field).

A tentative list of items to be considered:

[5] $\phi_i = -\log_2(d_i)$; $\bar{\phi} = \sum_{i=1}^n \phi_i f_i$; $d_{50} = 2^{-\bar{\phi}}$

- parameterization BBL: test SG model, implement Soulsby-model, connect to sediment suspension; investigate model sensitivity to parameters such as bed roughness, drag coefficient etc.;
- bathymetry: change slopes,and maximum and minimum depth to be resolved;
- tidal forcing: vary prescribed forcing magnitude at deep water;
- wind forcing : test various strengths and directions;
- wave fields: vary characteristics (T_p, H_s), direction; test regimes of wave dominance $\leftrightarrow$ current dominance $\leftrightarrow$ current only;
- paramterization interior eddy viscosity/diffusivitiy: test/compare different models available (analytical expression, bottom-KPP, ...)
- sediments: consider 1 size class, 2 classes, test for various sizes; test dependence of results on parameters such as critical shear stress τ_{cs}, resuspension parameter γ_0; consider the effect of depletion limitation in the bed;
- test resolution vertical: how well can we resolve the vertical concentration profile within reasonable cpu-time? is it feasable to even resolve nepheloid layer?
- test sensitivity to resolution in horizontal and time;
- ...

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