# ROMS time stepping flow chart

- rho_eos :

  → Compute the potential density anomaly ${\rho'_1}^n = \rho_{\text{EOS}}(T^n, S^n)$

  → Compute compressibility coefficients ${q'_1}^n\,(T^n, S^n)$

$${q'_1}^n = 0.1\left[\rho_0 + {\rho'_1}^n\right] \cdot \frac{K_0^{\text{ref}} - K_0(T^n, S^n)}{(K_{00} + K_0(T^n, S^n)) \cdot \left(K_{00} + K_0^{\text{ref}}\right)}$$

  → Compute Brunt-Väisälä frequency $\text{bvf}^n({\rho'_1}^n, {q'_1}^n, z)$

  → Compute vertically integrated and vertically averaged densities ${\rho^*}^n$ and $\bar{\rho}^n$

$$\bar{\rho}^n = \sum_{k=1}^{N} \bar{\rho}_k^n \text{Hz}_k^n / \sum_{k=1}^{N} \text{Hz}_k^n \qquad \text{where } \text{Hz}_k^n = z_{k+\frac{1}{2}}^n - z_{k-\frac{1}{2}}^n$$

- set_HUV : Compute volumetric fluxes

$$\text{HUon}_k^n = \text{Hz}_k^n \text{ on\_u } \text{u}_k^n \qquad [\text{on\_u} = \Delta\eta]$$

- omega : Compute volumetric fluxes in the vertical direction (via the continuity equation)

$$\text{W}_{k+\frac{1}{2}}^n = -\sum_{k'=1}^{k} (\text{div HUon})_{k'}^n + \frac{z_{k+\frac{1}{2}}^n - H}{\zeta^n - H} \sum_{k=1}^{N} (\text{div HUon})_k^n$$

- prsgrd : Compute horizontal pressure gradient via a density Jacobian method

$$\text{ru}_k = \left.\frac{\partial \text{P}^n}{\partial x}\right|_z$$

- rhs3d : Compute right hand side for 3D momentum equations at time $n$

  → Add in Coriolis and curvilinear transformation terms;

$$\text{ru} = \text{ru} + \text{Coriolis}$$

→ Add in horizontal advection of momentum (QUICK-scheme);

$$\mathrm{ru} = \mathrm{ru} + \text{2D advection}$$

→ Add in vertical advection terms (parabolic splines reconstruction);

$$\mathrm{ru} = \mathrm{ru} + \text{vertical advection}$$

Start computation of the forcing terms for the 2D (barotropic mode) momentum equations

$$\mathrm{rufrc} = \sum_{k=1}^{N} \mathrm{ru}_k$$

- pre_step3d : predictor step on u,v,Hz,t

$$\mathrm{Hz}^{n+\frac{1}{2}} = \left(\frac{1}{2}+\gamma\right)\mathrm{Hz}^n + \left(\frac{1}{2}-\gamma\right)\mathrm{Hz}^{n-1} - (1-\gamma)\Delta t \cdot [\mathrm{div\ HUon}^n + \mathrm{div\ W}^n]$$

Horizontal advection (UP3 scheme)

$$q^{n+\frac{1}{2}} = \left(\frac{1}{2}+\gamma\right)\mathrm{Hz}^n q^n + \left(\frac{1}{2}-\gamma\right)\mathrm{Hz}^{n-1} q^{n-1} - (1-\gamma)\Delta t \cdot \mathrm{div}_h\ (\mathrm{HUon}^n q^n)$$

Vertical advection (centered fourth-order scheme with harmonic averaging)

$$q^{n+\frac{1}{2}} = \frac{1}{\mathrm{Hz}^{n+\frac{1}{2}}}\left[q^{n+\frac{1}{2}} - (1-\gamma)\Delta t \cdot \partial_z\left(W^n q^n\right)\right]$$

$$u^{n+\frac{1}{2}} = \frac{1}{\mathrm{Hz}^{n+\frac{1}{2}}}\left[\left(\frac{1}{2}+\gamma\right)\mathrm{Hz}^n u^n + \left(\frac{1}{2}-\gamma\right)\mathrm{Hz}^{n-1} u^{n-1} - (1-\gamma)\Delta t \cdot \mathrm{ru}\right]$$

Boundary conditions on $q^{n+\frac{1}{2}}$ and $u^{n+\frac{1}{2}}$; $\quad u^{n-1} = \mathrm{Hz}^n u^n$

correction of $u^{n+\frac{1}{2}}$ to ensure that

$$\sum_{k=1}^{N} \mathrm{Hz}_k^{n+\frac{1}{2}} u_k^{n+\frac{1}{2}} = \frac{3}{2}\,\mathrm{DU\_avg1} - \frac{1}{2}\mathrm{DU\_avg2} \qquad \mathrm{DU\_avg1} = \left\langle \overline{U} \right\rangle^n; \mathrm{DU\_avg2} = \left\langle\left\langle \overline{U} \right\rangle\right\rangle^{n-\frac{1}{2}}$$

$$\mathrm{HUon}_k^{n+\frac{1}{2}} = \mathrm{Hz}_k^{n+\frac{1}{2}}\ \mathrm{on\_u}\ \mathrm{u}_k^{n+\frac{1}{2}}$$

- step2d : barotropic time stepping ($\Delta\tau$ = barotropic time step)
  barotropic step loop

$$\text{Drhs} = D^{m+\frac{1}{2}} = h + \left[\left(\frac{3}{2}+\beta\right)\text{zeta}^m - \left(\frac{1}{2}+2\beta\right)\text{zeta}^{m-1} + \beta\text{zeta}^{m-2}\right]$$

$$\text{urhs} = \overline{u}^{m+\frac{1}{2}} = \left(\frac{3}{2}+\beta\right)\text{ubar}^m - \left(\frac{1}{2}+2\beta\right)\text{ubar}^{m-1} + \beta\text{ubar}^{m-2}$$

$$\text{DUon} = \overline{U}^{m+\frac{1}{2}} = \text{dn\_u}\ D^{m+\frac{1}{2}}\ \overline{u}^{m+\frac{1}{2}}$$

$$\text{zeta\_new} = \zeta^{m+1} = \zeta^m + \Delta\tau\ \text{div}\ \overline{U}^{m+\frac{1}{2}}$$

$$\text{Dnew} = D^{m+1} = h + \zeta^{m+1}$$

$\longrightarrow$ Compute time-averaged fields

$$\text{Zt\_avg1} = \text{Zt\_avg1} + a_m \zeta^{m+1}$$

$$\text{DU\_avg2} = \text{DU\_avg2} + b_m \overline{U}^{m+\frac{1}{2}}$$

$$\zeta' = \delta\ \zeta^{m+1} + (1-\delta-\gamma-\epsilon)\zeta^m + \gamma\zeta^{m-1} + \epsilon\zeta^{m-2}$$

$\longrightarrow$ rubar = vertically integrated pressure gradient $\mathcal{F}(\zeta')$

$\longrightarrow$ rubar = rubar + horizontal advection

$\longrightarrow$ rubar = rubar + coriolis term

$\longrightarrow$ rubar = rubar + bottom drag

$\longrightarrow$ First 2D time step : rufrc = rufrc − rubar, rubar = rubar + $\mathcal{F}$

$$\text{DUnew} = (D\overline{u})^{m+1} = D^m \overline{u}^m + \Delta\tau\ (\text{rubar} + \text{rufrc})$$

$$\overline{u}^{m+1} = \text{DUnew}/D^{m+1}$$

$$\text{DU\_avg1} = \text{DU\_avg1} + b_m \text{DUnew dn\_u}$$

$\longrightarrow$ Last 2D time step : update vertical coordinate system [set_depth]

$$z_{k+\frac{1}{2}} = z^{(0)}_{k+\frac{1}{2}} + \text{Zt\_avg1}\left[1 + \frac{z^{(0)}_{k+\frac{1}{2}}}{H}\right]$$

update Hz    end barotropic step loop

- set_HUV2 :

$$\text{Correction of } \mathrm{u}^{n+\frac{1}{2}} \text{ to ensure that } \sum_{k=1}^{N} \mathrm{Hz}\ \mathrm{u}^{n+\frac{1}{2}} = \mathrm{DU\_avg2}$$

$$\mathrm{HUon} = \mathrm{Hz}\ \mathrm{u}^{n+\frac{1}{2}}$$

- omega : Compute volumetric flux $W^{n+\frac{1}{2}}$ for vertical velocity
- rho_eos : Compute density anomaly $\rho(T, S, z)^{n+\frac{1}{2}}$
- prsgrd : Compute horizontal pressure gradient

$$\mathrm{ru} = \frac{\partial \mathrm{p}^{n+\frac{1}{2}}}{\partial \mathrm{x}}$$

- rhs3d : Compute right hand side for 3D momentum equations at time $n + \frac{1}{2}$

- step3d_uv1 :

$$\mathrm{u}^{n+1} = \mathrm{u}^{n} + \Delta t\ \mathrm{ru}^{n+\frac{1}{2}}$$

- step3d_uv2 :

  solve the tri-diagonal problem due to the implicit treatment of vertical viscosity

$$\text{Correction of } \mathrm{u}^{n+1} \text{ to ensure that } \sum_{k=1}^{N} \mathrm{Hz}\ \mathrm{u}^{n+1} = \mathrm{DU\_avg1}^{n+1}$$

  set lateral boundary conditions for $u^{n+1}$

  2D/3D coupling :

$$\mathrm{ubar} = \frac{\mathrm{DU\_avg1}^{n+1}}{\sum_{k=1}^{N} \mathrm{Hz}_k^{n+1}}$$

  compute mass fluxes through grid box faces at time $n + \frac{1}{2}$

$$\mathrm{u}^{\star} = \frac{1}{2}(\mathrm{u}^{n+1} + \mathrm{u}^{n})$$

$$\text{Correction of } \mathrm{u}^{\star} \text{ to ensure that } \sum_{k=1}^{N} \mathrm{Hz}\ \mathrm{u}^{\star} = \mathrm{DU\_avg2}$$

$$\mathrm{HUon}^{n+\frac{1}{2}} = \mathrm{Hz}\ u^{\star}$$

- omega : Compute volumetric flux $W^{n+\frac{1}{2}}$ for vertical velocity
- step3d_t : advance the tracers to $n+1$

$$q^{n+1} = \mathrm{Hz}^n q^n - \Delta t \ \mathrm{div}\left(\mathrm{FlxU}^{n+\frac{1}{2}} q^{n+\frac{1}{2}}\right)$$

$$q^{n+1} = q^{n+1} - \Delta t \ \mathrm{div}\left(\mathrm{W}^{n+\frac{1}{2}} q^{n+\frac{1}{2}}\right)$$

$\rightarrow$ Add vertical diffusion

$$q^{n+1} = \frac{q^{n+1}}{\mathrm{Hz}^{n+1}}$$

Set lateral boundary conditions for $q^{n+1}$